C4 Differential Equations

1. June 2010 qu. 8

(i) Find the quotient and the remainder when $x^2 - 5x + 6$ is divided by $x - 1$. **[3]**

(ii) (a) Find the general solution of the differential equation $\left(\dfrac{x-1}{x^2-5x+6}\right)\dfrac{dy}{dx} = y - 5.$ **[3]**

(b) Given that $y = 7$ when $x = 8$, find y when $x = 6$. **[4]**

2. Jan 2010 qu. 10

(i) Express $\dfrac{1}{(3-x)(6-x)}$ in partial fractions. [2]

(ii) In a chemical reaction, the amount x grams of a substance at time t seconds is related to the rate at which x is changing by the equation

$$\frac{dx}{dt} = k(3-x)(6-x),$$

where k is a constant. When $t = 0$, $x = 0$ and when $t = 1$, $x = 1$.

(a) Show that $k = \dfrac{1}{3}\ln\dfrac{5}{4}$. [7]

(b) Find the value of x when $t = 2$. [4]

3. June 2009 qu. 9

A tank contains water which is heated by an electric water heater working under the action of a thermostat. The temperature of the water, θ° C, may be modelled as follows. When the water heater is first switched on, $\theta = 40$. The heater causes the temperature to increase at a rate k_1 °C per second, where k_1 is a constant, until $\theta = 60$. The heater then switches off.

(i) Write down, in terms of k_1, how long it takes for the temperature to increase from 40 °C to 60 °C. [1]

The temperature of the water then immediately starts to decrease at a variable rate $k_2(\theta - 20)$°C per second, where k_2 is a constant, until $\theta = 40$.

(ii) Write down a differential equation to represent the situation as the temperature is decreasing. [1]

(iii) Find the total length of time for the temperature to increase from 40 °C to 60 °C and then decrease to 40 °C. Give your answer in terms of k_1 and k_2. [8]

4. Jan 2009 qu. 9

A liquid is being heated in an oven maintained at a constant temperature of 160 °C. It may be assumed that the rate of increase of the temperature of the liquid at any particular time t minutes is proportional to $160 - \theta$, where θ°C is the temperature of the liquid at that time.

(i) Write down a differential equation connecting θ and t. [2]

When the liquid was placed in the oven, its temperature was 20 °C and 5 minutes later its temperature had risen to 65 °C.

(ii) Find the temperature of the liquid, correct to the nearest degree, after another 5 minutes. [9]

5. June 2008 qu. 7

(i) Show that, if $y = \operatorname{cosec} x$, then $\dfrac{dy}{dx}$ can be expressed as $-\operatorname{cosec} x \cot x$. [3]

(ii) Solve the differential equation

$$\frac{dx}{dt} = -\sin x \tan x \cot t \quad \text{given that } x = \tfrac{1}{6}\pi \text{ when } t = \tfrac{1}{2}\pi.$$ [5]

6. Jan 2008 qu. 8

Water flows out of a tank through a hole in the bottom and, at time t minutes, the depth of water in the tank is x metres. At any instant, the rate at which the depth of water in the tank is decreasing is proportional to the square root of the depth of water in the tank.

(i) Write down a differential equation which models this situation. [2]

(ii) When $t = 0$, $x = 2$; when $t = 5$, $x = 1$. Find t when $x = 0.5$, giving your answer correct to 1 decimal place) [6]

7. June 2007 qu. 8

The height, h metres, of a shrub t years after planting is given by the differential equation

$\dfrac{dh}{dt} = \dfrac{6-h}{20}$. A shrub is planted when its height is 1 m.

(i) Show by integration that $t = 20\ln\left(\dfrac{5}{6-h}\right)$. [6]

(ii) How long after planting will the shrub reach a height of 2 m? [1]

(iii) Find the height of the shrub 10 years after planting. [2]

(iv) State the maximum possible height of the shrub. [1]

8. Jan 2007 qu. 9

(i) Find the general solution of the differential equation $\dfrac{\sec^2 y}{\cos^2(2x)}\dfrac{dy}{dx} = 2$. [7]

(ii) For the particular solution in which $y = \dfrac{1}{4}\pi$ when $x = 0$, find the value of y when $x = \dfrac{1}{6}\pi$. [3]

9. June 2006 qu. 5

A forest is burning so that, t hours after the start of the fire, the area burnt is A hectares. It is given that, at any instant, the rate at which this area is increasing is proportional to A^2.

(i) Write down a differential equation which models this situation. [2]

(ii) After 1 hour, 1000 hectares have been burnt; after 2 hours, 2000 hectares have been burnt. Find after how many hours 3000 hectares have been burnt. [6]

10. Jan 2006 qu. 8

(i) Solve the differential equation $\dfrac{dy}{dx} = \dfrac{2-x}{y-3}$,

giving the particular solution that satisfies the condition $y = 4$ when $x = 5$. [5]

(ii) Show that this particular solution can be expressed in the form $(x-a)^2 + (y-b)^2 = k$, where the values of the constants a, b and k are to be stated. [3]

(iii) Hence sketch the graph of the particular solution, indicating clearly its main features. [3]

11. June 2005 qu. 9

Newton's law of cooling states that the rate at which the temperature of an object is falling at any instant is proportional to the difference between the temperature of the object and the temperature of its surroundings at that instant. **A** container of hot liquid is placed in a room which has a constant temperature of 20 °C.

At time t minutes later, the temperature of the liquid is θ °C.

(i) Explain how the information above leads to the differential equation

$$\frac{d\theta}{dt} = -k(\theta - 20),$$

where k is a positive constant. [2]

(ii) The liquid is initially at a temperature of 100 °C. It takes 5 minutes for the liquid to cool from 100°C to 68°C. Show that

$$\theta = 20 + 80e^{-\left(\frac{1}{5}\ln\frac{5}{3}\right)t}.$$ [8]

(iii) Calculate how much longer it takes for the liquid to cool by a further 32 °C. [3]